COUL

Coulomb Balance

revised January 30, 2006

(*You will do two experiments; this one and the Electric Potentials and Fields. Sections will switch rooms and experiments half-way through the lab.*)

Learning Objectives:

During this lab, you will

1. communicate scientific results in writing.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. learn how to fit a power-law model to a set of experimental data.
4. learn how to use the value of chi-squared to determine if data support a model.
5. test a physical law experimentally.

A. Introduction

We have all observed effects of static electricity, such as the attraction of a comb for hair on a dry day or the spark when you touch a doorknob after walking across a carpet. However, it can be difficult to make a quantitative measurement of these effects. The forces involved are very small and the charges have a tendency to leak away while measurements are being made, particularly if there is much humidity in the air.

In this experiment you will test important aspects of Coulomb's Law, which describes these electrostatic forces. To overcome the problems described above, you will use one of the most sensitive instruments for measuring small forces, the torsion balance, while a high voltage power supply will be used to replenish the charge throughout the experiment.

A full report worth 60 points is required for this laboratory experiment.

B. Theory

B.1. Electrostatic Force

The magnitude of the mutual electrostatic force between two point charges Q_0 and Q_1 separated by a distance r is given by Coulomb's Law

$$F = k\frac{Q_1 Q_0}{r^2} \qquad (1)$$

where $k = 1/(4\pi\varepsilon_0)$ and ε_0 is the permittivity of free space. (*In the SI system of units which we use in this lab,* $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N{\cdot}m^2$.)

The mutual electrostatic force between two *insulating* spheres carrying uniformly distributed charges Q_0 and Q_1 is also expressed by Eq. 1, where r is the separation of the centers of the spheres. However, in this experiment we use conducting spheres and the presence of each charged sphere will distort the charge distribution on the other. For two spheres bearing charges of the same sign, the charges repel one another, moving towards the back of each sphere. You can think of this as increasing the effective separation of the charge distributions, leading to a reduction of the repulsive force. This reduction in the force is a function of the separation r of the spheres and their radii a, and is given approximately by the correction factor

$$f = 1 - 4(a/r)^3 \qquad (2)$$

so the force becomes

$$F_{Coulomb} = \left[1 - 4\left(\frac{a}{r}\right)^3\right]\frac{kQ_0Q_1}{r^2} \qquad (3)$$

B.2. The Coulomb Balance

The Coulomb Balance is illustrated schematically in Figure 1 (on the next page). A photograph of the balance, with labels attached to some of the major controls, is posted on the lab web site; examining this photo may make it easier for you to understand the following discussion.

The Coulomb Balance is a form of torsion balance and consists of a fine fiber A

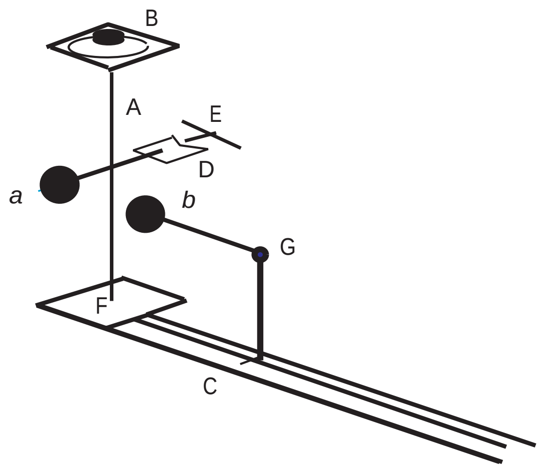

Figure 1: Coulomb Balance: *a* and *b* are the charged spheres, A - torsion fiber, B - adjustment knob with angular scale, C - linear scale, D - counterweight and damping vane, E - damping magnet and index line, F - zero adjustment, G - thumbscrew.

which supports sphere *a* and a counter weight and damping vane *D* mounted on a rod. When sphere *b* exerts a force on sphere *a*, the rod rotates about the fiber until the torque produced by the electrostatic force is balanced by the rotational torque exerted by the fiber. We can restore the rod to its original position by rotating the fiber support with the knob on the top of the device. You can then read the angle of rotation on the angular scale *B*.

A torsion balance is the rotational analog of the spring balance. Instead of the spring restoring force $F = -kx$, there is a restoring torque $\tau = \kappa\theta$. The magnitude of the torque τ is related to the Coulomb force acting between the spheres by the equation $\vec{\tau} = \vec{L} \times \vec{F}$ where $\vec{L}$ is the moment arm of the balance, *i.e.*, the distance from the torsion wire to the center of the sphere mounted on the torsion arm. You will measure θ but this is equivalent to measuring F, since θ is proportional to τ, and τ is proportional to F. It is not necessary to know the proportionality constants κ and L to do the analysis requested for this lab. You do need to realize that the angle through which the balance rotates is reduced by the correction factor described earlier, so that you need to correct θ to $\theta' = \theta/f$, where f is given in Eq. 2.

B.3. Electric Potential and Capacitance

We use a potential of 6 kV to charge the spheres. Later this semester you will prove that a conducting sphere can be treated as a spherical capacitor with a capacitance $C = 4\pi\varepsilon_0 a$, where a is the radius of the sphere. You will also learn that the charge on a capacitor is related to its potential by the formula $Q=CV$. Thus, the charge on the sphere should be

$$Q = CV = 4\pi\varepsilon_0 aV. \qquad (4)$$

Therefore, when you charge the conducting spheres to a particular voltage, you will have placed a certain charge on each of the spheres.

C. Apparatus

This experiment requires only the Coulomb torsion balance and a high voltage power supply. You will have to judge the precision of the analog voltage meter from its scale. The radii of the spheres that you will charge are 1.90 cm with negligible uncertainty.

D. Procedures

D.1. Precautions

The Coulomb Balance is very delicate. Do not change its orientation on the table. To avoid any static charge from disrupting the measurements, you should remain as far as possible from the spheres. Do not touch the spheres with your hands. Touch the supporting rods only if an adjustment is necessary. Any moisture or oils from your hands will increase the rate of charge leakage from the device. Avoid breathing on the spheres as the moisture in your breath will upset the measurements. Make measurements as quickly as possible, before the charge leaks away.

If your clothing creates static electricity, remove it (*within the bounds of decency*) and keep it away from the apparatus. Nylon coats, for example, should be kept at the back of the room. Turn off your computer monitor since the screen can accumulate static charge. Do not move about the room any more than necessary as air currents will disturb the balances. Make a complete set of measurements from a single sitting position so that your body capacitance doesn't change your experimental conditions.

Your power supply produces several thousand volts, but only microamperes of current, so while they can give you a shock, like touching a doorknob after walking across a carpet in winter, it probably can't kill you. However, you should not remove or replace the probes and you should avoid touching the electrodes. Turn on the supplies only for the brief period while you are charging the spheres. (*If the probes are not plugged into the power supply or if the grounding wire from power supply to apparatus is not connected, ask a TA for help*.)

D.2 Technique

The most difficult aspect of this lab for most people is taking measurements quickly enough. Assume that you have 5 seconds from the time you charge the spheres until enough charge leaks off to affect your measurements. Some hints to help you work quickly:

1. Charge the sphere on the torsion wire first. This gives it time to settle down while you charge the sphere that slides along the linear scale.
2. Touch the sphere that is suspended on the torsion wire in a manner that minimizes any subsequent oscillation. This means that you should NOT touch it on its side near or opposite the other sphere. Try touching it from above or below or on the side opposite the damping magnets.
3. You don't have to readjust the torsion knob to zero before each measurement. If you've already made one measurement and know where the next one is likely to end up, leave the knob in that position. This will lead to much less oscillation.
4. Don't strive for perfection in aligning the indices on the balance. As you wait for perfection, the spheres will discharge and the balance will drift back to zero. Incorporate your estimate of the average imperfection in alignment into your error estimate for the angle of the torsion wire setting.
5. Your apparatus includes a black probe that lets you ground the spheres. Grounding of the spheres is only necessary if you want to decrease their charge. It is not necessary to ground them to recharge them to the same or a higher value. You can ground them between each measurement if you wish but this will slow you down a bit and shouldn't change any results.

Air currents from the building ventilation system can upset the Coulomb Balances. When the first group announces that it is ready to take careful data (*after perfecting your technique with a few rough measurements*), the instructor will turn the thermostat up and leave it up until the last group is finished taking data. To further minimize air currents, the instructors will generally not walk around the room to spot problems. If you are having trouble, raise your hands and/or your voices.

D.3. Alignment

Make sure the vane is free to move between the two magnets. (*The magnets supply eddy current damping of the motion, something that may be explained in lecture later this semester.*) In order to minimize parallax, the index line on the stand should be just above that on the vane and should be read straight-on, not from the side of the balance. Move the sphere that slides on the linear scale as far as possible from the torsion balance. Remove any charge from both spheres by touching each with the black grounding wire. Then rotate the large, calibrated top knob *B* on the apparatus until the angle scale reads zero. Carefully rotate the plastic zero adjustment cylinder *F* (***but don't loosen its thumbscrew***) at the bottom of the fiber until the vane index is properly aligned with the fixed index mark.

Slide sphere *b* until it just touches sphere *a*. Be sure the vane index remains aligned. Read the scale. It should read 3.8 cm, the diameter of the spheres. If necessary, adjust this by loosening the thumbscrew *G* that locks in place the rod with the sliding sphere *b* and moving this sphere as necessary.

E. Measurements

You will take two sets of data. For the first set, you will measure how the repulsive force changes with different amounts of charge on the spheres, controlling the charge by controlling the voltage. For the second set of data, you will keep the voltage and charges fixed but measure the variation in the force as a function of the distance between the spheres.

E.1. Variation of Force with Charge

Turn on the power supply. Slide sphere *b* to its maximum separation from sphere *a*. Touch the black ground probe to each sphere in turn to ground it. Set the power supply to 3 kV. Touch the red high-voltage probe briefly to each sphere to charge it. Turn off the power supply. Work quickly to complete the following steps so that each measurement is finished before the charge leaks off the spheres.

Slide sphere *b* to 8 cm on the linear scale, being cautious not to jar the torsion suspension, rotate the knob to balance the torque, and record the angle measurement. This is not as easy as it sounds. Note that sphere *a* and the wire form a heavily damped torsion pendulum with a long period so that you need to exercise judgment about when it stops moving. Rotate the knob to the angle you think you will need and wait about 5 seconds for the motion to settle. Try to estimate the central position of the oscillation before readjusting the knob. You must compromise between precision in determining when the setup is in balance and allowing the charge to leak away, which will lead to a continuous shift in the balance adjustment back towards zero.

Repeat this first measurement a few times until you are familiar with the process and can produce consistent results. You must separate the spheres as much as possible before each measurement to ensure that the correct charge is placed on them. If you try to recharge them in their final positions, you will be placing the wrong charge on them due to the correction factor f in Eq. 2. Record at least 5 good measurements, average them for the analysis requested later and consider the spread in their values when you estimate the error in your data. You will probably see that the uncertainty in your measurement of θ is more than the resolution of the scale on the torsion balance knob. Likewise, the uncertainty in measuring the distance r between the spheres is more than just the resolution of the scale for the sliding sphere. This uncertainty should include an estimate of the uncertainty in the 'final' position of the sphere mounted on the balance as it swings to and fro while you are measuring θ.

Next, move sphere *b* to maximum separation, set the supply potential to 6 kV and charge the spheres. Changing the charging voltage will also change the charge placed on the spheres. Set the spheres to 8 cm separation and measure the balance angle. Make 5 measurements.

E.2. Inverse-square Law

Set the output voltage to 6 kV and measure force versus separation, beginning with sphere *b* at 20 cm, then at 18, 16, 14, 12, 10, 8, 7, 6 and 5 cm. Before each measurement, move sphere *b* to its maximum separation from sphere *a* and recharge both spheres. You are not required to take 5 readings at each of these distances; take as many as you think appropriate.

F. Analysis

Normally you are expected to include error bars for all data and plots. You should include error estimates for every value of r and θ given in your report but, because of the nuisance in its calculation, you need only calculate the error in θ' once, for your 6 kV data at r = 12 cm. You may then use this same value for all your θ' error estimates. (*However, your TA may give a bonus for correctly propagating error for every value of* θ'.)

F.1. Force as a Function of Charge

Compare the forces exerted with 3 kV and 6 kV charging potentials. Assume the charge on each sphere is proportional to the charging potential, as indicated by Eq. 4, and determine whether the force is directly proportional to the square of the charge, as predicted by theory. (*Note that this doesn't require the calculation of the force or the charge, just a proper set of ratios. You may use the fact that the force is directly proportional to the angle* θ *of the torsion knob. You can use* θ *rather than* θ' *here because the correction factor f should divide out in your analysis as both r and a are the same in these two measurements. In your Theory section, you should show that the Eqs. 1 and 4 imply that ratio of the forces, and thus the angles, is equal to the ratio of the square of the voltages. Also, remember that you cannot make a determination of whether or not data support a theory or not without uncertainties! Also recall from mechanics lab that your abstract should quote all numerical results.*)

Although it's not necessary to know that actual charge on the spheres to complete your tests of Coulomb's Law, calculate this charge anyway. Use Eq. 4 to calculate the charge on a sphere for V = 6 kV. (*Since this quantity won't be used anywhere, you do not need to calculate its uncertainty; however, you should only quote it to a reasonable number of significant figures.*)

F.2. Force as a Function of Distance

You are asked to do two different types of analysis of your data. The Basic Analysis section has you reformat your data so that the expected plot should appear as a straight line. This is convenient since your eye is better at judging straight lines than it is at judging more complex curves. Also, in the absence of calculating power, fits to straight lines are easier to perform. There is no question, though, that the Advanced Analysis is a more sophisticated technique.

F.2.1. Basic Analysis

Since the angle θ is proportional to the restoring torque, and therefore to the force, we can analyze θ (*actually* θ') versus $1/r^2$ to see how well the inverse-square law is obeyed. In *Origin*, make columns of r and θ. Set the values in a new, third column to $x = 1/r^2$ and set the values in a fourth column to $\theta' = \theta/f$, where f is the correction factor of Eq. 2. If you don't rename your columns and if your distances were collected in cm, your formula to do this should look like:

COL(B)/(1-4*(1.9/COL(A))^3).

Plot θ' vs. $1/r^2$, and fit a straight line to the graph. How good is the fit? Is the expected linear relation obeyed? Report and comment on the value of the **chi-squared per degree of freedom** $\left(\chi^2_{\mathrm{DOF}}\right)$ (See Appendix VIII, page 2). The value of χ^2_{DOF} is an indicator of the extent to which the data support the model. If you used "Fit Linear" in *Origin* with proper error bars on your values of θ', then the value "SD" that *Origin* reports is $\sqrt{\chi^2_{DOF}}$.

F.2.2 Advanced Analysis

Use the *Non-linear Curve Fit* routine in *Origin* to check whether or not the exponent really is 2 in the inverse square law. You will fit your data to a curve in the form of $\theta' = A/r^p$ where A is a constant (*which should be related to* $Q_0Q_1/4\pi\varepsilon_0$) and p gives the power of the power law (*which we expect to equal 2*)

Follow these steps:

1. Plot θ' vs. r. (*Scatter plot*)
2. Select ANALYSIS / NON - LINEAR CURVE FIT / ADVANCED FITTING TOOL. (*If the window for this routine pops up showing a MORE button, press it; otherwise you are left in a BASIC mode and can't see all of the necessary commands.*)
3. Select FUNCTION - NEW.
   - Type: USER-DEFINED
   - Uncheck the "USE ORIGIN C" box
   - Form: EXPRESSION
   - Click box: USER DEFINED PARAMETERS
   - Parameter names: A, p
   - Indep. var: x
   - Dep. var: y
   - Definition: A/x^p
   - [Do not type "y=".]
4. Select OPTIONS-CONTROL.
   - Weighting Method: ARBITRARY DATASET.
   - Select your *y*-error bars from the list under "AVAILABLE DATASETS"
5. Select ACTION - FIT
   - If a box pops up asking you which data to use for this fit, select ACTIVE DATASET
6. For starting values, set A=5000 and p=2.
7. While performing the following steps, watch the curve that should appear on your plot and keep an eye on the values of A and p in their windows. Click 1 ITER (*for 1 iteration of the calculation*). If this works, click *100 ITER* and repeat until the values of A and p stabilize.

<u>Quote your final value of the exponent p and its uncertainty.</u> (*Recall from mechanics lab that your abstract should quote all numerical results.*) Your uncertainty should come from the uncertainty returned by *Origin's* least square fitting routine for the value of p. To calculate this uncertainty properly, it is necessary to know the uncertainties in the values of θ' and r. <u>You should discuss these uncertainties in your report and include them as error bars in your *Origin* plots.</u>

The constant A is of no interest in this case since we have not calibrated the torsion fiber and will not try to measure its torsion constant, κ, or the permittivity of free space, ε_0. However, you could use your data to determine ε_0 if you wished. This determination would require calibrating the torsion constant, something that could be done directly using a system of weights and a pulley or indirectly by comparison to a linear spring.

There is one last thing you may try if you wish, but it isn't required. Try forcing *Origin* to fit your data with a curve p = 2. This will give you a good visual idea how close your data lies to the *accepted* formula. You can do this by returning to the curve fit menu and unchecking the box next to p under 'VARY?'. Run a few more iterations so that the value of A can be adjusted for a best fit, then plot this line as a dashed line to distinguish it from your previous fit.