EPF

Electric Potential & Fields

revised 26 January 2010

Learning Objectives:

During this lab, you will

1. communicate scientific results in writing.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. learn how to fit a non-linear model to a set of experimental data.
4. learn how to use the value of chi-squared to determine if data support a model.
5. test a physical law experimentally.

A. Introduction

In this experiment, you will map the electric potential and find the electric field for a two-dimensional system of conductors. You may not yet have seen in lecture all of the terms used in this section of the manual, but this should not prevent you from performing a perfectly fine experiment. In this lab, you should learn to appreciate electric fields, electric field lines, electric potentials, equipotentials and the relationships between these various quantities. They are all defined in your textbook as well as in the write-up below.

You will investigate several arrangements of electric conductors, geometries that are common in the world around us and that will be discussed in detail in the course lectures; among these are the pair of opposite charges, or dipole, the parallel plate capacitor and the hollow conductor.

You will write a full paper worth 60 points for this lab. There is graph paper at the end of this write-up for you to use for your EPF maps that will go with the paper.

B. Theory

B.1. Electrostatic Force

The magnitude of the mutual electrostatic force exerted between two charges Q_1 and Q_0 separated by a distance r is given by $F = kQ_1Q_0/r^2$, where $k = 1/(4\pi\varepsilon_0)$ and ε_0 is the permittivity constant.

The vector form of this expression for the force exerted on Q_1 by Q_0 is

$$\vec{F}_{10} = k\frac{Q_1Q_0}{r_{01}^2}\hat{r} \qquad (1)$$

where the unit vector $\hat{r}$ points from Q_0 to Q_1.

Figure 1 illustrates the repulsive force exerted on Q_1 by Q_0 when both charges have the same sign (+ *in Fig. 1*). An attractive force results when the charges have opposite signs.

Since $\vec{F}$ is a vector, the net force on a test charge produced by several charges must be found by computing the vector sum of the forces produced by the individual charges.

Note the similarity between the expressions for the gravitational and electrostatic forces:

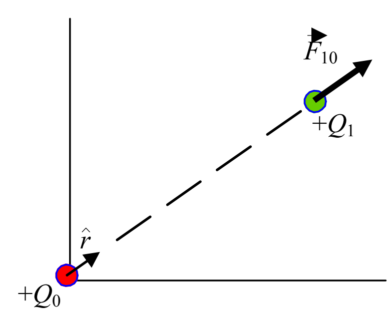


Figure 1: Repulsive force between like charges.

$$\vec{F}_g = G\frac{m_1 m_0}{r_{01}^2}\hat{r} \text{ and } \vec{F}_e = k\frac{Q_1 Q_0}{r_{01}^2}\hat{r}$$

Both are examples of "$1/r^2$ laws," so-named because the field strength varies as the inverse square of the distance from the source particle. There is however an important difference; since there are two kinds of charge, positive and negative, the electrostatic force can be either attractive or repulsive while the gravitational force is always attractive.

B.2. Electrostatic Field

The electrostatic field (or *electric field*) is defined at any point in space as the force per charge which would be exerted on a positive test charge Q_1 placed at that point. The electric field is measured in newtons per coulomb or, equivalently, volts per meter.

Equation 1 gives the force exerted on our test charge Q_1 resulting from the presence of charge Q_0. To determine the force *per charge* exerted by Q_0, we simply divide Eq. 1 by the test charge Q_1. We find

$$\vec{E} = \frac{\vec{F}}{Q_1} = k\frac{Q_1 Q_0}{Q_1 r^2}\hat{r} = k\frac{Q_0}{r^2}\hat{r} \qquad (2)$$

where the unit vector points outward from Q_0 and thus the field points outward from positive Q_0 or inward toward negative Q_0.

The force on a test charge Q_1 is

$$\vec{F} = Q_1 \vec{E}. \qquad (3)$$

Since $\vec{F}$ is a vector, it follows that $\vec{E}$ must also be a vector and the resulting field produced by a group of charges must be found by vector addition.

There are corresponding expressions for the gravitational field; near the surface of the earth we express the gravitational field as $\vec{g}$ and calculate the gravitational attraction of the earth as $\vec{F} = m\vec{g}$.

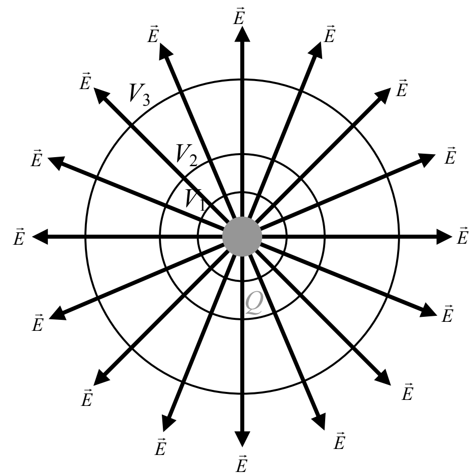


Figure 2: Field lines and equipotential surfaces from a point charge.

B.3. Electric Field Lines

It's very difficult to depict an electric field by drawing a lot of electric field *vectors*. Instead, electric fields are generally illustrated using electric field *lines*. Conceptually, electric field lines are created by joining up electric field vectors but this must be done in a way that preserves information about the strength and direction of the field. The direction of the field lines is parallel to $\vec{E}$ at any given point in space, which means that the field lines are always perpendicular to any equipotential lines. The strength of the field is proportional to the density of the field lines. This latter property means that one can't just randomly select which field lines should be drawn.

The electric field lines from a point charge are radial, pointing outward from a positive charge (*as illustrated by the radial lines in Figure 2*) and inward toward a negative charge. The lines are also evenly spaced around a point charge since the field strength should not depend on direction.

To draw field lines in practice, one can start at each charge and draw a set of short, equally spaced lines emanating from that charge. The lines are equally spaced because, very near a point charge, the field of that point charge must dominate the field of other charges and so is uniform in direction. The number of lines should be proportional to the strength of the charge since the density of the field lines must be proportional to the strength of the field. If two charges are of equal strength they should have the same number of field lines. A reasonable number of field lines for our purposes might be 8 (spaced 45 degrees apart) or 12 (spaced 30 degrees apart).

Then just focus on each line in turn and continue drawing it as it moves away from the charge, keeping each line perpendicular to the equipotential surfaces, which also guarantees that the field lines are parallel to the $\vec{E}$ vectors. Because of the symmetry of the problems in this lab, many of the lines that start on one conductor should terminate on an analogous line at another conductor. A few lines may just travel to the edge of our universe (*the water tray*).

Normally when you finish drawing field lines, you put an arrow on each one to indicate that the field actually points from positive to negative charges. However, in this lab you won't have charges or even simple DC potentials, so there is no real fixed direction to indicate.

B.4. Electric Potential

The difference in electric potential between two points in space, labeled *a* and *b*, is the negative of the work done by the $\vec{E}$-field, per charge, when a unit test charge is moved from *a* to *b*:

$$\Delta V_{ab} = -\int_a^b \vec{E} \bullet d\vec{l} \qquad (4)$$

Electric potential is measured in *volts*.

For a field created by one or many point charges, it is common to discuss absolute potentials by defining them with respect to a reference point at infinity. The electric potential at a point *r* is then the negative of the work done when a unit test charge is moved from an infinite distance, where the field is essentially zero, to some closer, finite distance, *r*. Eq. 4 leads to

$$V = \frac{kQ}{r} \qquad (5)$$

for the potential at a distance *r* from a single point charge *Q*, analogous to the gravitational potential $V = GM/r$ of a single point mass.

Potential is a scalar so the potential produced by several charges is just the algebraic sum of the potentials produced by each single charge. The only sign comes from the sign of the charge.

B.5. Electric Equipotential Surfaces

Because the electric potential of a point charge varies inversely as the distance from the charge, all points in space the same distance from the charge have the same potential, as long as there are no other charges in the neighborhood. It follows that the contours of constant potential (*or the equipotentials*) are spheres in 3-dimensional space, centered on the point charge.

A 2-dimensional drawing through the point charge would show circular equipotentials as illustrated in Fig. 2. The equipotentials produced by groups of particles, such as the dipole described in the next section, are smooth surfaces in 3 dimensions and smooth curves in 2 dimensions. Equipotential surfaces never intersect, since the potential at some point can't have two different values.

B.6. Measuring Electric Fields

The most direct way to measure an electric field $\vec{E}$ in space would be to place a small test charge $+q$ at various points and to determine the direction and magnitude of the force $\vec{F}$ on the charge. However this would be a very difficult experiment to carry out.

An alternate method is to measure the electrostatic potential V (*V is defined in Section B.4*) with a voltmeter. The electric field can easily be calculated if the potential is known. The relation between $\vec{E}$ and V is:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \quad (6)$$

The derivatives in this formula are partial derivatives, which you may not have encountered yet. However, if you know that the field points in one given direction, say the x-direction, Eq. 6 simplifies to

$$\vec{E} = -\frac{dV}{dx}\hat{i} \quad (7)$$

where the derivative in Eq. 7 is a 'normal' derivative.

The potential V that appears in Eq. 6 can be viewed in terms of the equipotential surfaces discussed above. In general, there are an infinite number of such surfaces, each with a different value of V, but one can describe an electric field very well by measuring and drawing a carefully chosen selection of equipotential surfaces.

From Eq. 6, the strength of the electric field is proportional to the negative of the rate of change of the electric potential. If you draw a representative sampling of equipotential surfaces, say at 1 volt intervals, then the electric field is largest where these surfaces are most closely spaced.

Eq. 6 can also be used to prove that the direction of the field is always perpendicular to the equipotential surfaces. For example, if the equipotential surfaces are all parallel to the yz plane, then the electric field vector will point in the x-direction as in Eq. 7.

Mapping electric fields in this fashion is similar to drawing weather fronts (*isobars or lines of equal pressure*) or mapping the topography of the earth. A topographic map shows lines of constant elevation, equally spaced in altitude. Where the lines are closest together, the terrain is steepest. If you travel along a line, you stay at the same height. The fastest change in height (or the steepest *grade*) is perpendicular to a line since this takes you most quickly to a neighboring line. In fact, the operation described in Eq. 6 is known as taking the *gradient* of the potential.

B.7. Electric Dipoles

An electric dipole consists of equal positive and negative charges, $+Q$ and $-Q$ separated by a distance d. (*There is no analogy in gravitation since the gravitational force is always attractive.*) A characteristic dipole field is only observed at points a distance $r >> d$; for points very close to one charge or the other, the contribution of the nearest charge will dominate.

The electric *field* produced by the dipole can be calculated by *vector* addition of the fields produced by the two individual charges. The electric *potential* of the dipole can be calculated by *scalar* addition of the individual potentials.

For two point charges with charges $+Q$ and $-Q$ on the x-axis at positions $+a$ and $-a$ respectively, Eq. 5 implies that the potential along the x-axis will be

$$V = kQ\left[\frac{1}{|a-x|} - \frac{1}{|a+x|}\right]. \quad (8)$$

See Figure 3.

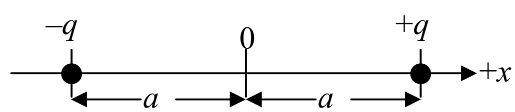


Figure 3: Dipole geometry.

For the region between the charges in Figure 3, Eq. 2 implies that the electric field along the x-axis will be

$$\vec{E} = -kq\hat{i}\left(\frac{1}{(a-x)^2}+\frac{1}{(a+x)^2}\right). \quad (9)$$

B.8. Parallel Plates

Two infinitely long conducting sheets with opposite charge densities will produce a uniform electric field between the sheets (pointing from the positive sheet to the negative sheet) and no electric field outside the sheets. It follows that the potential between the sheets will linearly increase from the negative sheet to the positive sheet and the potential outside the sheets will be constant.

B.9. Conductors

A conductor is a material in which there are charge carriers that can move freely. If an electric field is applied to a conductor, the charge carriers will very quickly redistribute themselves until the field is canceled and the electric forces on the charge carriers falls to zero. It follows that no static electric field exists within a conductor. It also follows that a conductor defines an equipotential volume; the interior and surface are all at the same potential. (*Internal charges do move when potentials are first applied to a conductor but this motion ceases very quickly, in about* 10^{-15} *s.*)

C. Apparatus

Electrical potentials in space are difficult to measure because normal instruments (*and people*) are good enough conductors to disrupt the potentials one is trying to measure. For example, you might not realize that the electric potential near the Earth's surface increases at about 100 V/m. This makes sense if you realize that lightning at 1 million volts comes from clouds about 10 km high. Do you sense a 200 V potential difference between your toes and your nose? Do you get a shock when you stand up after getting out of bed? Of course not! The reason is that your body is a very good conductor compared to the air around it and so your mere presence eliminates the potential difference in your vicinity. Even a very good DMM will have the same effect.

However, if the potential is created in a medium that is itself a fairly good conductor, such as tap water, the potential is easy to measure. If we map and draw equipotentials, we can then also draw the electric field lines; they are just curves perpendicular to the equipotential surfaces at every point.

The fields you will map are created in rectangular plastic trays filled with tap water. Figure 4 shows the three different field geometries you will examine. You will use various brass conductors to set the geometry of the field and an AC (*alternating current*) transformer to generate it. The transformer is nominally rated for 12 V AC

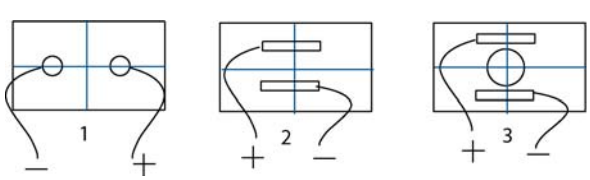


Figure 4: Field geometries.

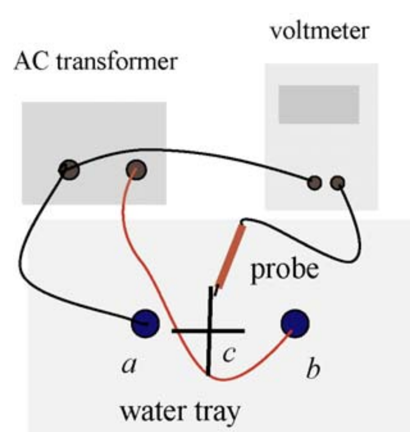


Figure 5: EPF setup.

but may actually produce 16 V AC or more. A sheet of graph paper and a DMM are used for quantitative mapping. The setup is illustrated in Figure 5.

D. Measurements

D.1. Procedure

A photograph of the first setup is posted on the lab web site (although you may want to reverse the colors, red for black, of the leads plugged into the DMM in that picture). Place a sheet of metric-ruled graph paper (major divisions of 1 cm with 1 mm sub-divisions) beneath the water tray to provide a coordinate system (unless a sheet is already in place). Make sure the origin of the coordinate system is at least roughly near the center of the tray and that the axes are approximately parallel to the edges of the tray. Check that the tray is filled with water to a depth of about 1.0 ± 0.5 cm, and place two cylindrical electrodes in the tray symmetrically with their centers at $(x,y) = (\pm 7,\ 0)$ cm as shown in Figures 3 - 5. (Be careful - there may be marks on the graph paper under your tray which are **not** at this spacing.) Note that if we divide the tray into four quadrants, we can use symmetry arguments to predict the potential distribution in the entire tray from careful measurements made in only one quadrant.

Connect the two electrodes to the output terminals of the 12 V AC transformer. Connect the common (black input) of your DMM to the left or '*a*' electrode and use a probe plugged into the red V/Ω input to make your measurements of potential. Be certain to hold this probe **vertical** when making measurements or you will distort the reading. Set the DMM on "AC" volts and use an appropriate scale for measuring 12 V. Although your DMM can provide 4 or 5 significant figures for your measurements, you need only read V to ± 0.1 volts for this lab; striving for precision beyond this is difficult, time-consuming and will not help illustrate the basic phenomena of fields and potentials.

Touch the probe to each electrode, *a* and *b* and record each potential (V_a, V_b). Now place the probe midway between the two electrodes and measure the potential V_c.

D.2. Symmetry

Since the pattern should be symmetric in the four quadrants, you need only make careful measurements in a single quadrant such as quadrant 4 in Fig. 5 (*except for the measurements required along the x- and y-axes, which cut across all four quadrants*.)

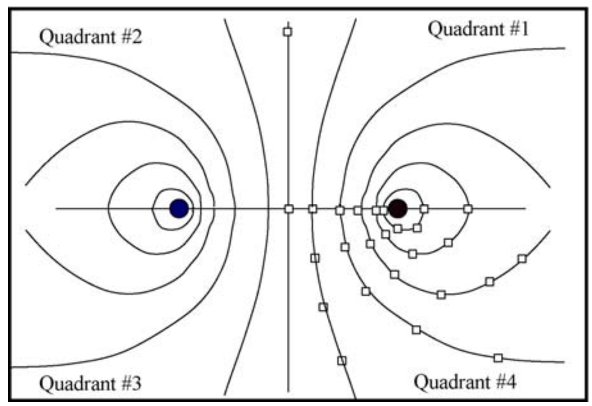


Figure 6: Measurements for dipole equipotentials.

D.3. Dipole Map

The square blocks in Figure 6 represent the measurements that you are asked to make. <u>Take a set of measurements along the entire *x*-axis</u>, from one edge of the water tray to the other. Record values of potential as a function of position every $\Delta V = 1.5$ volts from the value at point *c* at the origin. You will be asked later to use this data to calculate the electric field as a function of position along the *x*-axis. (*You may find in this and in later measurements that it is difficult or impossible to take data very close to a conductor. You should simply skip such* *measurements, ignoring individual points and even entire equipotential lines when this data cannot be acquired reliably*.)

Make an *Origin* plot of this data, with a fit to the theoretical model. Use the *Non-linear Curve Fit* routine in *Origin* to check whether or not the potential varies as Eq. 8. You will fit your data to a curve in the form of $V = B\left[\frac{1}{|a-x|} - \frac{1}{|a+x|}\right] + C$ where B is a constant (*which should be equal to kQ*), a is the distance each electrode is from the origin, and C is the potential at infinity. (Eq. 8 assumes 0, but we have taken our zero point to be the left electrode.)

Follow these steps:

1. Plot V vs. x. (*Scatter plot.*)
2. Select ANALYSIS → FITTING → NON - LINEAR CURVE FIT → OPEN DIALOG.
3. Click on the CREATE/EDIT FUNCTION button.

    Click NEW FUNCTION.

    Indep. var: *x*

    Dep. var: *y*

    Enter Parameter names: *a*, *B*, *C*

    Select EQUATION from the FUNCTION FORM drop menu.

    Function:

    y = *B**(1/abs(*a*–*x*) - 1/abs(*a*+*x*)) + *C*

    PARAMETER SETTINGS: For starting values, set a=7, B = 10, and C = 6 (if you entered x-values in *Origin* in centimeters, otherwise use 0.07, 0.1, and 0.06.)

    Name your function and select SAVE. Click OK and a different window will pop up.

4. Select DATA SELECTION.

    Click on the plus to expand INPUT DATA, then click on the plus for RANGE 1 and the plus for y.

    Weighting Method: ARBITRARY DATASET.

    Select your *y*-error bars from the list under "DATA"

5. Select 1-ITER (which two buttons left of the FIT button)
6. While performing the following steps, watch the message that appears in the lower part of this dialog window. If fit converged, click the button to the left of the FIT button, and then click FIT.

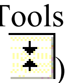
Report your value of chi-squared per degree of freedom (χ^2_{DOF}). Compare your fitted value of *a* to what you measured. To calculate this value properly, it is necessary to know the uncertainties in the values of V and x. You should discuss these uncertainties in your paper and include them as error bars in your *Origin* plots. Recall that a χ^2_{DOF} of about one means that the model fits the data well; a value of much greater than one means that the model does not fit the data well.

Next, take a set of measurements along the entire *y*-axis. Make these measurements every 5 cm from the origin. Principles of symmetry imply that these values should be identical to each other; the *y*-axis is an equipotential line. If you do **not** find this to be the case, consult with your TA as there may be something wrong with your apparatus or with your experimental technique.

Now take all of the data you will need to make a detailed mapping of selected equipotential lines in quadrant 4, noting that you should already have the data for an equipotential line along the *y*-axis. Record this data in your lab notebook numerically, rather than plotting it immediately, with each point being an x, y co-ordinate pair for a given voltage. This procedure makes it easier to analyze your data in some depth later, if necessary. Use the value of the *y*-axis equipotential as a reference and take data so that you can map out other equipotentials in the fourth quadrant with a spacing of 1.5 V, 3.0 V, 4.5 V, etc. compared to this reference. You should take enough points, at least 3 or 4, along each equipotential line to ensure that you can make a reliable estimate of its shape. (*You should be able to take this data fairly quickly. Start with your probe at the electrode and move it out in a straight line. Record the position and potentials when the voltage is at 1.5 V, 3.0 V, etc. Continue this process until you've reached the edge of the quadrant, then return to the electrode and start on another line. About 6 lines should give you coverage of the entire quadrant.*)

D.4. Parallel Plates

Refer to Figure 3 to see the parallel plate arrangement. The brass bars should be parallel to the *x*-axis and centered about the *y*-axis with their **inner** edges placed at $y = \pm 6$ cm. (***Don't CENTER them on y = ±6 cm!***)

Start by taking a set of data along the entire *y*-axis (*including the region 'outside' the pair of brass bars*), recording values every 1.5 volts from the value at the origin. Plot this data in *Origin*. To test our model, you will need to do a linear fit to the data from *between* the plates. To restrict a fit to only part of your data, click the "data selector" button (on the Tools toolbar; on the left by default; look like ) and click-and-drag the two markers to the right and left extremes of data you want to fit. Perform a linear fit on the data, and report your value of χ^2_{DOF}.

Next, take a set of data along the entire *x*-axis, collecting points every 5.0 cm from the origin. This should be an equipotential line. Finally, take the data in the 4th quadrant required to construct a detailed map of the equipotential lines with spacings between lines in multiples of 1.5 volts from the value at the origin.

D.5. Hollow Cylinder between Parallel Plates

Refer to Figure 3 and place a hollow conducting cylinder midway between the two parallel plates. In addition to making a map of a quadrant for this setup, take readings every 0.5 cm along the *y*-axis between the bars (don't worry about readings outside the bars but do take readings inside the hollow cylinder). Plot this data in *Origin*. Take a few readings at random points inside the hollow cylinder but not on the *y*-axis; these may help you appreciate what happens inside a conductor.

E. Analysis

E.1. Equipotential Plots & Field Lines

Complete your maps of the equipotentials (*in all four quadrants, using symmetry to complete the lines in quadrants 1-3*) for the three setups. Sketch the corresponding electric field lines on the same maps, using as guidance the principles laid out in section B.3. (*Use the graph paper supplied at the end of this write-up for these maps. You only need to make three maps but we have included extra paper in case you make a mistake.*)

E.2. Electric Field

The magnitude of the electric field can in principle be determined from Eq. 6 but in practice this is difficult to do without analytic equations for the equipotential surfaces and some experience with partial derivatives. One can instead find ΔV, the potential difference between adjacent contours, and divide by Δr, the separation of the contours, to estimate the magnitude of the electric field

$$E = -\Delta V/\Delta r. \qquad (10)$$

The direction of the field is simply perpendicular to the equipotential surfaces. You should be able to estimate the magnitude and direction of the field anywhere from your equipotential plots but you are only being asked to do this for the following situations.

1. Calculate the field as a function of position along the *x*-axis for your dipole data. In this case, Eq. 10 reduces to

$$E = -\Delta V/\Delta x \qquad (10A)$$

Since you need to compare two readings of potential, *V*, to find $\vec{E}$, your calculations of $\vec{E}$ will give the average field between those points where you measured *V*. [*This means that if you measured, say* V = *3.0 volts at* x = *2.0 cm and* V = *5.0 volts at* x = *5.0 cm, you should write that you calculated E at* x = *3.5 cm to be*
E = - (*5.0 - 3.0) V* / (*0.050 - 0.020*) *m or*
E=-67 volts/m .]

You can do this calculation manually or you can do it with *Origin*, since your data should already be in a table in *Origin*. To do it in *Origin*, you need to create two new columns and set their values. It is important to realize that the symbol I refers to a row in a table of data (see Appendix IV: *Origin*) so the values of potential require a formula something like: (COL(B) - COL(B)[I - 1]) / (COL(A) - COL(A)[I - 1]), assuming that you have your x values in column A and your potentials in column B. You will also need to calculate the positions as: (COL(A) + COL(A) [I-1])/2 . Note that you must also restrict the rows to which these formulae apply to the second through the last, since there is no data in row 0.

Plot the electric field as a function of position. Fit Eq. 9 to the data between your electrodes. Follow the same plotting procedure that was outlined on page 7, making the appropriate changes in the equation. (Hint: How does equation 9 compare with equation 8 which you plotted already?) Report your value of χ^2_{DOF} .

2. Next, calculate the field along the *y*-axis for the parallel conductor setup (only for the setup **without** the hollow cylinder).

3. What is the average field, including error bars, inside the hollow cylinder used in the last setup?

F. Appendix-Three Approximations

Three different approximations have been incorporated into this experiment. They are described here for completeness but you do not have to worry about them in your analysis.

1. You have not actually performed an electrostatic experiment but an analog to one. However, with current flowing through the water, the electric field lines are indeed perpendicular to the equipotentials, so the analog is good as long as the conducting medium (water) has uniform resistance. The brass electrodes have very low resistance relative to water, so the potential may be considered to be constant over an entire electrode.

2. We use an AC potential to reduce electrolytic effects. A DC current would cause rapid plating of minerals from our Cleveland tap water onto the electrodes. We could use purer water but would discover that pure water is a very poor electric conductor. The use of 60 Hz AC is acceptable because the frequency of the electric signal is low enough that the field changes slowly compared to the movement of the charges. So the experiment you have conducted yields the same results as a DC potential would have provided.

3. The two cylinders placed in the water tray are described as representing two point charges. This is not true. In fact, they represent two parallel line charges, positioned

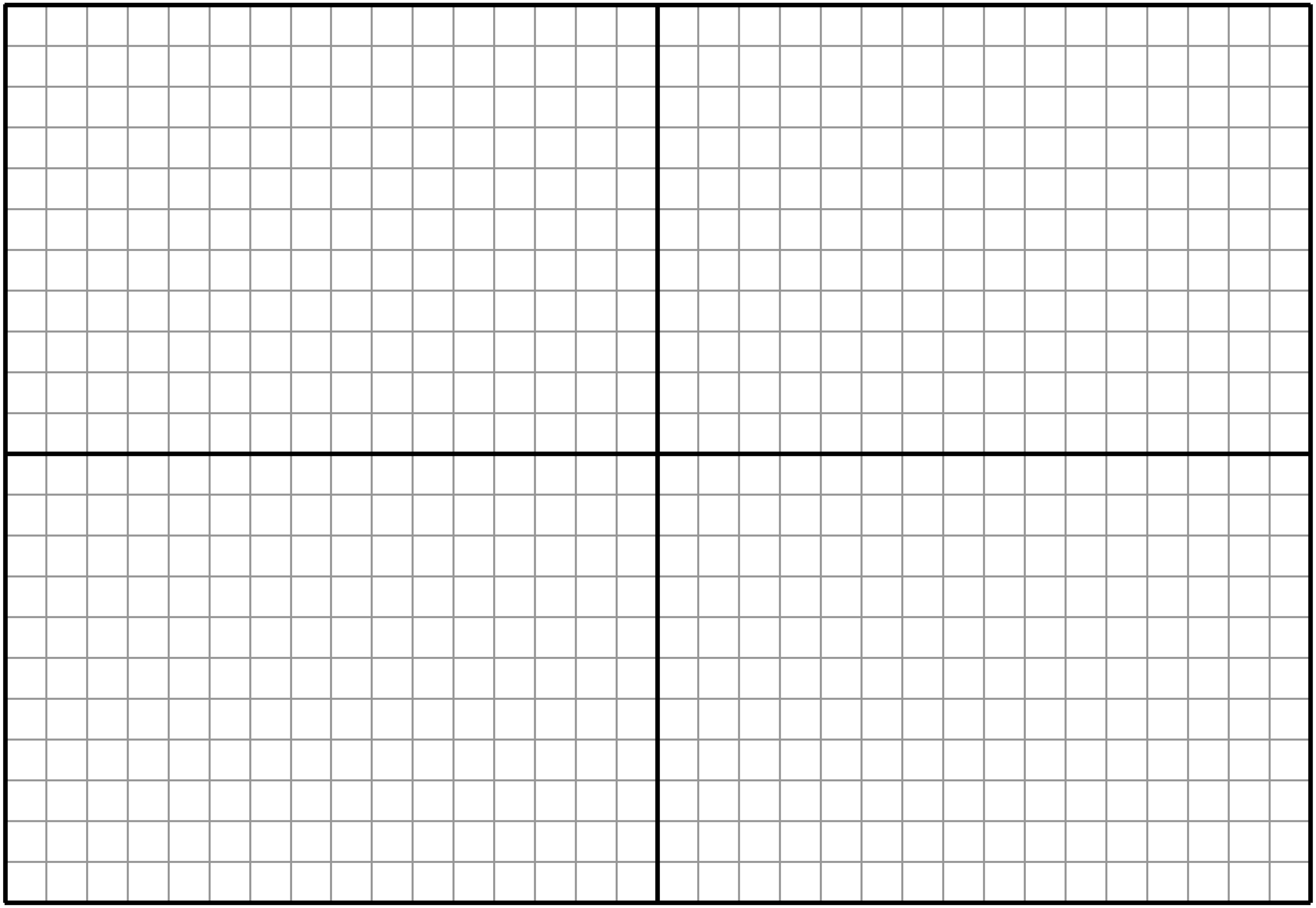
with their axes perpendicular to the tray. The reason is similar to the reason that the parallel brass bars are said to represent parallel plates, which is true, rather than parallel line charges resting in the tray. The water tray is a 2-D slice of a 3-D world and whatever conductors we place in the tray should be imagined to be a 2-D slice of an infinitely tall 3-D object.

Fortunately, the difference in the potentials of a dipole and a pair of parallel line charges is subtle as long as you don't stray too far from the charges. You probably discovered that the careful data you took along the *x*-axis for the "dipole" wasn't fit very well by our dipole model. (*Your TA may give you a bonus if you correctly determine the theoretical potential between two oppositely charged line charges, fit that model to your data, and determine if the model fits your data.*)

If you wanted to use water to simulate two point charges, you'd have to suspend two conducting spheres in a tank, rather than a tray, of water.

Figure ____ : Electric Potential and Field Map of ________________________

Each square represents 1 cm



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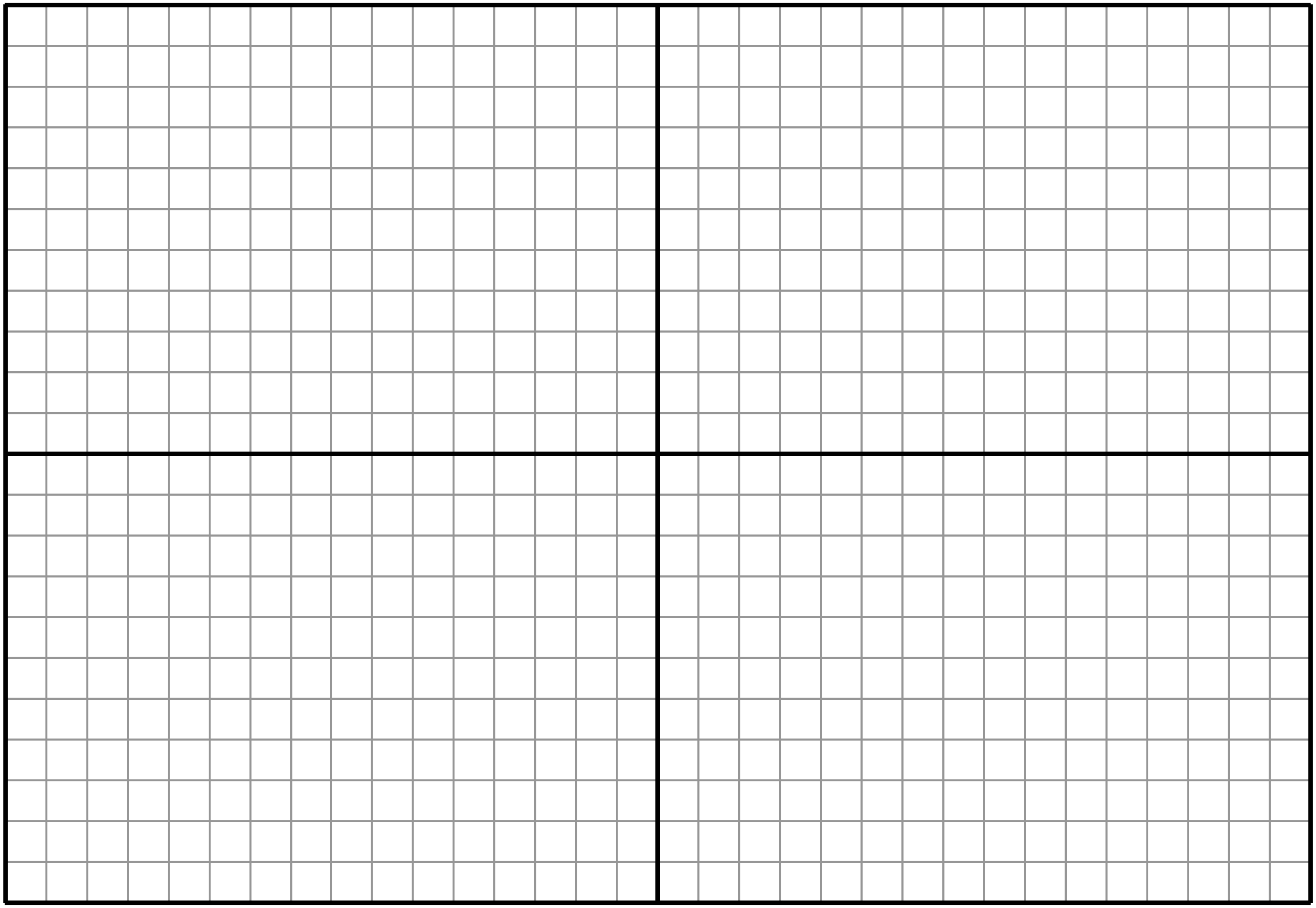

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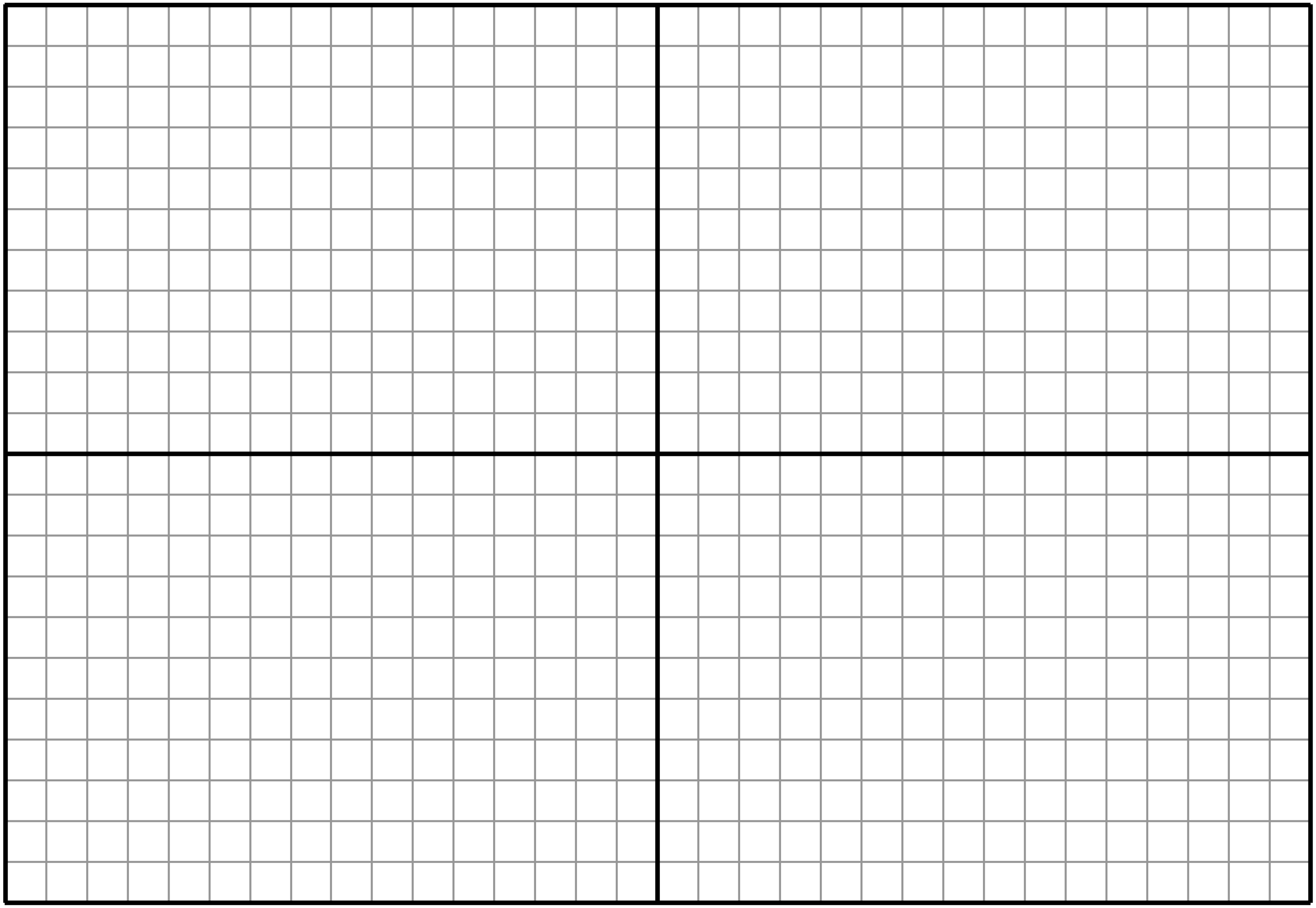

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Each square represents 1 cm

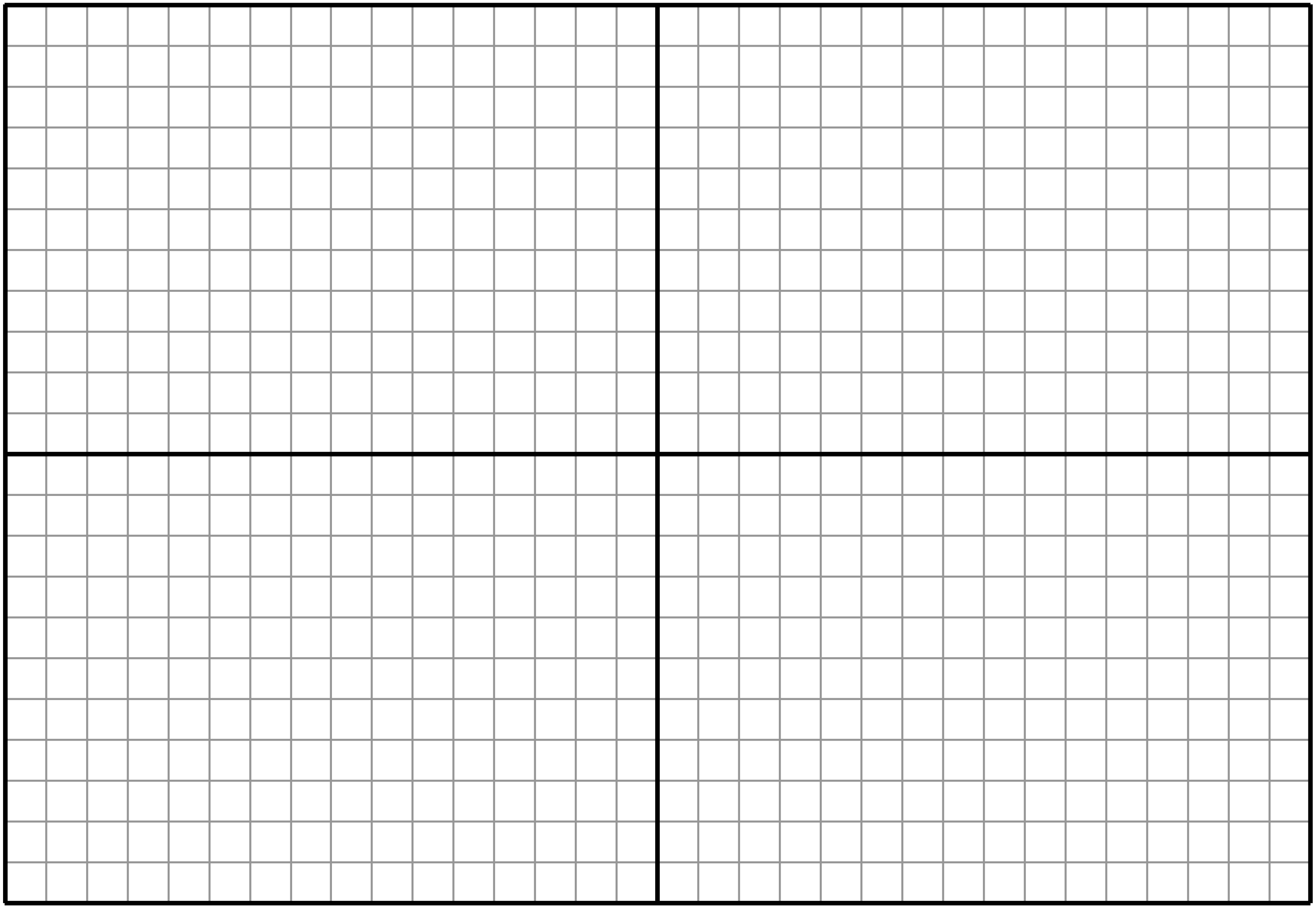

Figure ____ : Electric Potential and Field Map of ________________________

Each square represents 1 cm