CME

Conservation of Mechanical Energy

revised June 9, 2009

Learning Objectives:

During this lab, you will

1. learn how to communicate scientific results in writing.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. be introduced to the concept of relative (or fractional) change.
4. be introduced to a technique for accounting for a systematic error.
5. test a physical law experimentally.

A. Introduction

Conservation laws play an important role in physics. In classical physics, quantities such as energy, linear and angular momentum and the amount of electric charge are conserved. The list of conserved quantities expanded significantly during the development of modern physics. For example, the total number of heavy elementary particles, or baryons (*including protons and neutrons*), appears to be constant in the universe. The same rule applies to the number of light particles, or leptons (*such as the electrons, muons and neutrinos*). Quantities named parity and strangeness are also conserved while relativity requires expanding the law of conservation of energy to include the equivalence of mass and energy in the form of Einstein's famous law, $E = mc^2$.

Physicists place such confidence in these laws that they have postulated the existence of new elementary particles to explain apparent failures of conservation laws in carefully performed experiments. No violation of the law of conservation of energy has ever been substantiated.

Because energy takes many forms, it is easy to believe that it has been lost or gained in an isolated experiment. For example, a rolling cart coming to a halt suggests that kinetic energy has disappeared from the system. However, the kinetic energy of the rolling cart may have been transformed into gravitational potential energy or into thermal energy. Similarly, a high-energy photon may disappear, reappearing as a low-energy particle-antiparticle pair. Part of the kinetic energy of the high-energy photon has been converted into the rest-mass energy of the particle-antiparticle pair through Einstein's relation.

In this experiment, you make two types of measurements:

a. conversion of gravitational potential energy of a falling body into the kinetic energy of the body and a cart to which it is tied,
b. conversion of gravitational potential energy into the potential energy stored in a stretched spring.

You must hand in a paper for this lab worth 45 points. Detailed guidance for writing this paper is at Section F of this write-up.

B. Apparatus

You will use a PASCO track with encoded pulley, a low-friction cart, weight hanger and weights. You will also use a block, a spring and a spirit level. Data will be recorded using a computer running the *Logger Pro* program.

C. Theory

Several equations are useful in this experiment:

Kinetic energy: $K = \frac{1}{2}mv^2$ (1)

Gravitational potential energy:

$$U_g = mgh \qquad (2)$$

Potential energy of a stretched spring:

$$U_k = \frac{1}{2}kx^2 \qquad (3)$$

Hooke's Law: $F = -kx$ (4)

We will assume that the pulley is massless and frictionless for this experiment.

D. Gravitational Potential Energy

D.1. Procedure

Weigh the cart and its load, then place your loaded cart near the middle of the track. Connect one end of the string to the cart and pull it over the encoded pulley. The arrangement is shown in Figure 1.

Using the spirit level, check that the track is level. If necessary, make the track level using the knob at the end of the track. Note whether the track appears to have a slight curve in it.

While the friction of the cart is small, it is not negligible for this experiment and it is necessary to compensate for this effect. This can be done by hanging small masses (*paper clips*) on the string so that the gravitational force on these masses equals the force of kinetic (*not static*) friction on the cart. To do this, you must hang a weight then give the cart a push towards the pulley. Adjust the number of paper clips until the cart moves with constant speed, indicating that there is no net force on the cart. Check whether the same number of paper clips balances the force due to friction regardless of the speed of the cart. You may use *Logger Pro* to help you identify a nearly constant speed. After starting the program, go to the file menu and open *P:Logger Pro 3__Mech Labs\CME.*

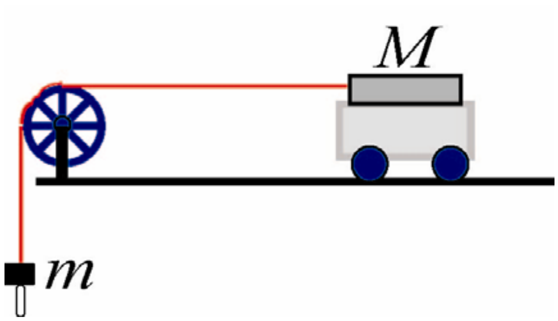

Figure 1: Cart with Counterweight

Measure and record this mass and an estimate of its uncertainty; justify the estimate of the uncertainty. (This uncertainty is probably larger than the resolution of the scale.)

Think carefully about whether (and how) you should include this mass in your energy conservation calculations and describe your conclusions in your paper. Note that the potential energy of this mass changes as the cart moves (*because the height of the masses change*) while its kinetic energy changes only if the mass and cart accelerate or decelerate. The energy lost to friction should be a function only of the distance the cart (*and mass*) move.

Hang an additional mass of about 30 grams from the string. You can use your hanger or attach stock masses by bending one of your paper clip masses to double as a hanger. The stock masses are accurate enough that you may ignore any error in the values stamped on them.

Release the cart; it should accelerate. Use the *Logger Pro* program to record the motion of the cart as it is pulled along the track by the falling weight. If necessary, you may adjust the amount of hanging weight so that the cart accelerates smoothly along the track. Note that the string must always be taut between the cart and the weight hanger.

Repeat several times for practice. When you are confident of your experimental technique, record a set of measurements to be shared by your group for later analysis and save the data file to your folder on the L: drive.

D.2. Analysis

Note that in your analysis you include the energy loss due to friction, which was measured with paper clips. *Logger Pro* measures the time interval between successive spokes of the encoded pulley, each of which corresponds to a length of 1.5 cm.

From these measurements, you will calculate distance, velocity and acceleration.

Export your data from *Logger Pro* and into *Origin* using the FILE → EXPORT AS → TEXT command in *LoggerPro* and FILE → IMPORT → Simple Single ASCII in *Origin*. Delete the Time and GateState columns, then examine the data at the beginning and end of your file and delete any rows that are obviously corrupted by your releasing and catching the cart.

In *Origin,* add three additional columns; these will be used for kinetic energy (K), potential energy (U), and total energy (E). Your columns will contain, in order, $t, y, v, a, K, U,$ and E. You may find it convenient to rename or label your columns with these symbols. (*Right click on each column in turn, select PROPERTIES and rename the column or add a label.*) These labels simplify understanding what is in each column. Use y and v to calculate the total kinetic energy K and potential energy U of the system. Assume that the potential energy and the kinetic energy are both zero initially so that the initial total energy E_0 is zero. The potential energy becomes increasingly negative as the weight falls, while the kinetic energy becomes increasingly positive. Use the COLUMN → SET COLUMN VALUES command and enter appropriate formulae for K, U, and E, such as (*assuming you've renamed the columns*)

*0.5*M₁*col(v)^2* for K

*-M₂*col(y)*9.81* for U

and

col(K) + col(U) for *col(E)*

M_1 and M_2 are the masses in kilograms for K and U, <u>don't forget to consider whether M_1 and M_2 should include the mass of the paper-clips used to compensate for friction</u>. Also, remember to use MKS/SI units. (*Occasionally Origin will decide that it can't calculate the values for the columns. If this happens to you, you can create a new data sheet, copy the appropriate data from the imported data sheet, and paste it in the new data sheet. You will then be able to perform calculations on the data*.)

Plot on a single graph, as functions of position y, the kinetic energy K, the potential energy U and the total energy $E = U + K$. (*To plot multiple sets of data on one plot, use the PLOT → SYMBOL → SCATTER command to call up the SELECT COLUMNS box, select each set of data – one x variable and one y variable – that you wish to plot in turn and click on the ADD box to add each set to the plot in turn.* ***Getting Origin to fit the correct data (E) will be easier if you select the total energy in your first x-y pair***.) If mechanical energy is conserved, you should have $E = 0$.

Fit a straight line to your total energy plot and, from the slope, <u>record the energy change per meter ($\Delta E/\Delta y$) and its uncertainty</u>. You should first click on the DATA command on the title bar to see that the appropriate variable (TOTAL ENERGY, E) is the variable selected for fitting (i.e., it is the quantity listed at the bottom of this menu with a checkmark). Then use ANALYSIS → LINEAR → FIT LINEAR → OPEN DIALOG to find the slope B of the line and its uncertainty. Copy the fit parameters into your plot, cleaning them up as necessary, fix the labels on the plot axes and elsewhere as needed and add a title, including partners' names, to the plot. Save a copy of this Origin project to your folder and then <u>print copies of your graphs, with fit results, to include in your paper</u>. Using the uncertainty on the weight needed to balance friction, <u>estimate the uncertainty on the energy lost to friction</u>. <u>State your conclusions about energy conservation in this experiment.</u>

E. Spring Potential Energy

E.1. Procedure

LoggerPro is not used for this experiment. All measurements are to be made "by hand."

E.1.a. Hooke's Law

Hook one end of the spring onto the end of the track and use a piece of string to attach the other end of the spring to the mass holder, as illustrated in Figure 2. Measure the position of the end of the spring using the scale on the track. The absolute position of the 'end' of the spring is not critical; only changes in the spring's length are required for this experiment. You may therefore develop your own technique for determining this position. One possibility is to attach a piece of tape to the end of the spring, drawing a line on the tape to serve as a reference mark. You may also use the knot in the string as a mark. A ruler laid across the track may help you in your measurements. As always, describe the methods you use for your measurements as well as any reasoning behind using those methods. Estimate the uncertainties in your measurements and describe your reasoning for choosing those uncertainties.

Add 5 grams to the hanger. Measure the new position. Repeat this process until you have a total mass of 100 grams, or until the mass hanger hits the ground. You will use these data to determine the spring constant, k.

E.1.b. Potential Energy of a Stretched Spring

Attach a total mass of 75 grams (*including the 50–gram hanger*) to the string. Pull on the string to raise the mass just high enough so that the spring is relaxed, *i.e.*, at the equilibrium length it would have with no weight on it. Record this position using the track scale. Release the string and observe the motion. The data you will need to record is the maximum excursion of the spring from the relaxed position, how far it moves as the mass first drops towards the floor. Before recording any data, you should increase the amplitude of the motion as much as possible. To do this, increase the mass on the counterweight until the counterweight comes within several cm of hitting the floor.

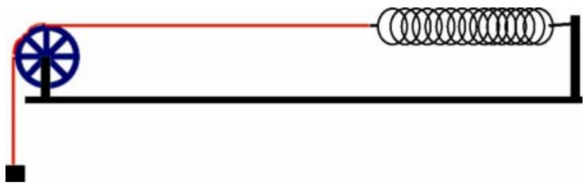

Figure 2: System with Added Spring

When you have a suitable mass and have perfected your release technique, start taking data. Record the maximum change in position of the end of the spring using the track scale. Each lab partner should record at least four measurements. Use the spread of the data to estimate the uncertainty in your measurement of the displacement.

In addition to recording the maximum change in position, also wait for the spring to stop oscillating and measure its final position.

E.2. Analysis

E.2.a. Checking Hooke's Law

Using *Origin,* make a graph of position versus hanging mass. Fit a straight line to the data and determine the spring constant k and its uncertainty from the slope of the line and Equation 4, which can be rewritten as

$$x = \frac{mg}{k} + x_0 \tag{5}$$

where x_0 corresponds to the (*unmeasured*) original length of the spring. Find the uncertainty in k from the uncertainty in the slope. Clean up, save and print a graph with your fit results to include in your paper.

E.2.b. Potential Energy

Consider the total energy of the system before the mass was released (U_k=0) and when it has reached its point of maximum excursion (U_g=0). Note that the kinetic energy does not appear in the total energy at these points; it is zero because the mass is at rest at both positions. Use the mean of your measurements of the change in position to compare the total energy of the system when the mass is in the two positions.

Determine how well energy is conserved in this experiment. Express your result in terms of the relative energy change (*the net change of the total energy divided by the change of gravitational energy*)

$$\varepsilon = \frac{\Delta U_k + \Delta U_g}{\left|\Delta U_g\right|} \qquad (6)$$

There are two sources of uncertainty (δ_ε) in ε: the uncertainty (δ_k) in the spring constant k and the uncertainty (δ_x) in your measurement of x. In your paper, explain why the uncertainty in hanging mass is negligible; that is, it gives a small contribution to the overall uncertainty in the energy measurements. Expand Equation 6 to display x and k explicitly and use the computational method or the derivative method to find the uncertainty in ε. To use the computational method calculate

i. $\delta_{\varepsilon,k} = |\varepsilon(k + \delta_k, x) - \varepsilon(k, x)|$
ii. $\delta_{\varepsilon,x} = |\varepsilon(k, x + \delta_x) - \varepsilon(k, x)|$

To use the derivative method calculate

i. $\delta_{\varepsilon,k} = \left|\frac{\partial \varepsilon}{\partial k}\right| \delta_k$

ii. $\delta_{\varepsilon,x} = \left|\frac{\partial \varepsilon}{\partial x}\right| \delta_x$

In either case, then calculate

iii. $\delta_\varepsilon = \sqrt{\delta_{\varepsilon,x}^2 + \delta_{\varepsilon,k}^2}$

It's also straight-forward to calculate the relative change in total energy from the time of release of the spring to the time when it stops oscillating. Calculate this relative change using Eq. 6. Was energy conserved? Explain.

F. Detailed Guidance for Paper

Here is detailed guidance on what should go into this lab paper. Future lab will not have this detailed guidance on the lab papers; the only guidance in addition to Appendix II you will have will be the underlining embedded in the manual.

This laboratory has two measurements: gravitational potential energy and spring potential energy. This fact leaves two logical ways of organizing the Introduction & Theory, Procedure, and Results & Analysis sections: 1) having both measurements (gravitational & spring) as sections, each with the appropriate subsections or 2) having each section with a sub-section for both measurements. We will adopt the first organization for the paper.

F.1. Abstract

It is generally best to write the abstract last, after you've completed the main body of the paper, so that you are certain of the points that you wish to highlight in the abstract. Although this lab has two parts, there will only be one abstract.

You should start this abstract with a sentence outlining the physical principle you tested in the laboratory. Then, spend 2-4 sentences describing the gravitational potential energy measurements, stating your value for $\Delta E/\Delta y$, and discussing any conclusions from that part of the experiment. Spend another 2-4 sentences describing the spring potential energy measurements, stating your value for ε, and discussing any conclusions from that part of the experiment. Finally, write a sentence with an overall conclusion.

F.2. Gravitational Potential Energy

F.2.a. Introduction and Theory

First develop the gravitational potential energy theory. List any assumptions you are making about the system. Your first equation should be

$$W_{nc} = \Delta K + \Delta U.$$

Substitute in appropriate expressions for ΔK and ΔU. Show that we can account for the energy dissipated by friction by adjusting the mass in ΔU by the amount necessary to cause the system to move with constant speed (i.e., whether and how you should include the mass of the paperclips.) State what value you expect the slope of a plot of total energy versus position to be. Make sure you define any symbols you use. You should have a figure of the apparatus either here or in the procedure. If you do not produce the figure yourself, make sure you properly acknowledge your source.

F.2.b. Procedure

Note: Do not write the Procedure section in the second person; you are not telling your reader how to do the experiment, you are telling your reader how **you** *did the experiment.*

Report the mass of the cart and its load with your estimate of the uncertainty, how you measured the mass, and your justification for the uncertainty. Say that you first found the force of friction on the system by finding the hanging mass required to cause the cart to move with constant velocity. State how you determined the cart moved at constant speed with the paperclips as counterweight, what the required mass to balance friction was, how you measured that mass, what your estimate of the uncertainty in that mass was, and what your reasons were for adopting that uncertainty. Say that you conducted the experiment by hanging an additional mass and recorded the system's motion. State the amount of additional mass you put on the end of the string and how you recorded the motion of the system after releasing it from rest.

F.2.c. Analysis

Say that you analyzed your gravitational potential energy data in *Origin* by plotting kinetic, potential, and total mechanical energy versus position. List the equations you used to determine kinetic, potential, and total energies using standard algebraic notation (**not** *Origin* syntax.) State which objects (cart, hanging mass, paperclips) were included in which mass. Attach the graph to the end of the paper and refer your readers to it. Report the slope of the best fit line to the total energy plot as a measurement interval. Determine the uncertainty in $\Delta E/\Delta y$ due to the uncertainty in the balancing mass. Add this uncertainty in quadrature with the uncertainty in $\Delta E/\Delta y$ due to the uncertainty in the slope and report $\Delta E/\Delta y$ as a measurement interval.

F.3. Spring Potential Energy

F.3.a. Introduction and Theory

Develop the spring potential energy theory. Start with Hooke's Law, rearrange it to express equilibrium position as a function of amount of hanging mass (with other constants.) Relate the slope of a plot of position versus mass to the spring constant (k) and other constants and solve the relation for k. Make sure you define any symbols you use.

Then refer to your first equation from the gravitational potential energy theory above and substitute in appropriate expressions for ΔK and ΔU (note that you have two kinds of potential energy in this measurement.) You should have a figure of the apparatus either here or in the procedure. If you do not produce the figure yourself, make sure you properly acknowledge your source.

F.3.b. Procedure

Say that your first step was to determine the k of the spring you were using by measuring the equilibrium position of the end of the spring with different amounts of hanging mass. State what method you used to define the "end" of the spring (e.g., tape at end of spring with line, knot on string, etc.), how you determined where the end was for the Hooke's Law experiment, what the uncertainty in that determination was, and why you adopted that value for the uncertainty. Report the range of mass tested.

Say that you then conducted the experiment by releasing the hanging mass from the position where the spring was relaxed and recording its position when the mass reached its lowest point. Report the total mass you used for this part of the experiment. State where the initial position was, how you determined that position, what the uncertainty in that position was, and why you adopted that value. State how you determined the maximum position, how many times you and your partner(s) repeated the experiment, what the value of the maximum position was, what your estimate in the uncertainty was, and your reasons for adopting that uncertainty. State your value for the final (equilibrium) position, the uncertainty in the final position, and your reasons for adopting that uncertainty.

F.3.c. Analysis

Say that you determined the spring constant from a plot of position versus hanging mass, attach a copy of the plot to your paper, and refer your readers to it. Report the slope of the best fit line as a measurement interval. Use the appropriate equation from your theory to determine the value of the spring constant, showing your calculations. Determine the uncertainty in k from the slope and any other uncertain quantities. (If you are going to treat one or more quantities as having negligible uncertainty, state your reason(s) for doing so.) Report k as a measurement interval.

Say that you took the zero point of gravitational potential energy to be the hanging mass' lowest point. Show your calculations of the total energy of the system in the two positions. Determine the relative energy change ε and show your calculations. Explain why the uncertainty in the hanging mass is negligible. State whether you will use the computational method or derivative method to determine $\delta_{\varepsilon,k}$ and $\delta_{\varepsilon,x}$. (You may use the same method for both or one method for one and the other method for the other as long as you tell your readers what you are doing.) Calculate δ_{ε} and show your work; report ε as a measurement interval. Calculate the relative energy change from the time of release of the spring to the time when it stops oscillating and its uncertainty. Report this relative energy change as a measurement interval.

F.4. Conclusions

Although we have two parts to this experiment, we will tie them together with a common conclusions section.

Compare your value for $\Delta E/\Delta y$ with the value expected from theory. State whether or not these values agree. If they agree, say that this experiment is consistent with theory and give a plausible source of random error. If they do not agree, give a plausible systematic error that has not been accounted for that might account for the disagreement. Give a possible way to reduce your plausible error.

Compare your value for ε with the value expected from theory. State whether or not these values agree. If they agree, say that this experiment is consistent with theory and give a plausible source of random error. If they do not agree, give a plausible systematic error that has not been accounted for that might account for the disagreement. Give a possible way to reduce your plausible error. State whether or not mechanical energy conserved from the time of release of the spring to the time when it stops oscillating. Explain this result.

F.5. Acknowledgements

You should acknowledge those who gave significant help in collecting data, preparing graphs, (*i.e., your lab partners*) or other areas of the paper that are of such a nature that they cannot be referenced with an endnote. You do not need to acknowledge help from the laboratory director or your teaching assistant. An example of an Acknowledgement section is in the sample lab paper.

F.6. References or Citations

You should clearly acknowledge the use of all external reference sources, including your textbook and laboratory manual. However, it is not necessary to cite help received from the laboratory director or your teaching assistant. The sample paper includes examples of citations.