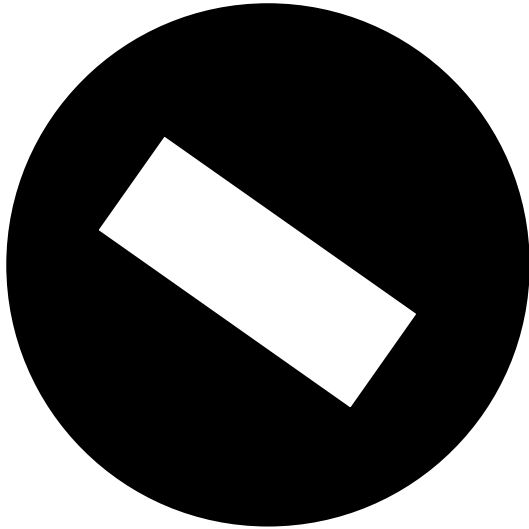


# UNC LAB: Error Analysis and Propagation Exercise

Revised September 6, 2005

Your Name: \_\_\_\_\_



This exercise is designed to help you understand error analysis and error propagation. You need to determine the area of the shaded region in the figure above; that is, the area of a circle minus the area of a rectangle. If the rectangle has a height,  $h$ , and width,  $w$ , and the circle has diameter  $d$ , then the shaded area is given by the formula  $A = \pi d^2/4 - hw$ .

Every measurement has an associated uncertainty. The uncertainties can be labeled with the symbol,  $\delta$ , which indicates a small change in the associated quantity. The uncertainties of  $h$ ,  $w$ , and  $d$  are given by  $\delta_h$ ,  $\delta_w$ , and  $\delta_d$  respectively.

Use a metric ruler to measure  $h$ ,  $w$ , and  $d$ , estimate the uncertainties  $\delta_h$ ,  $\delta_w$ , and  $\delta_d$  in your measurements of each quantity and enter these values

below, in cm. For your convenience, copy these values onto the other side of this page.

\_\_\_\_\_  $\pm$  \_\_\_\_\_ cm      \_\_\_\_\_  $\pm$  \_\_\_\_\_ cm      \_\_\_\_\_  $\pm$  \_\_\_\_\_ cm  
 $h$                      $\pm$                      $\delta_h$                      $w$                      $\pm$                      $\delta_w$                      $d$                      $\pm$                      $\delta_d$

Now calculate  $A = \pi d^2/4 - hw =$  \_\_\_\_\_  $\text{cm}^2$

To estimate the *uncertainty* in  $A$ ,  $\delta_A$ , we need to *propagate* each individual contribution to the uncertainty ( $\delta_h$ ,  $\delta_w$ , and  $\delta_d$ ) through the equation for  $A$  to find out how much each contributes to the uncertainty in  $A$  (*these terms are labeled as  $\delta_{Ah}$ ,  $\delta_{Aw}$ , and  $\delta_{Ad}$* ) and then add these contributions in quadrature  $\delta_A = (\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2)^{1/2}$ .

The first step is to determine  $\delta_{Ah}$ ,  $\delta_{Aw}$ , and  $\delta_{Ad}$ . This may be done by one of two methods. In the computational method, you calculate the change in  $A$  caused by substituting for each term, such as  $h$ , the value plus its estimated uncertainty, such as  $h + \delta_h$  (or  $h - \delta_h$ ). The derivative method has you calculate terms such as  $\delta_{Ah}$  using the idea that any small change in  $A$  due to a small change in  $h$  is given by the derivative of  $A$  with respect to  $h$ , treating all the other terms such as  $w$  and  $d$  as constants. This is properly called a *partial derivative* and uses the symbol  $\partial$  as in  $\frac{\partial A}{\partial h}$  rather than  $\frac{dA}{dh}$ . Once you know how  $A$  changes as a function of  $h$ , you can simply multiply this by the estimated uncertainty in  $h$ ,  $\delta_h$ , to find  $\delta_{Ah} = |\partial A/\partial h| \delta_h$ .

Now, for some practice in error propagation, fill in each of the blanks on the other side of this page.

## COMPUTATIONAL METHOD

$$\delta_{Ah} = |(\pi d^2/4 - hw) - (\pi d^2/4 - (h + \delta_h)w)| = \{ \text{this simplifies to } \delta_h w \} = \underline{\hspace{2cm}} \underline{\hspace{1cm}} \text{ (units)}$$

$$\delta_{Aw} = |(\pi d^2/4 - hw) - \underline{\hspace{2cm}} \underline{\hspace{1cm}}| = \underline{\hspace{2cm}} \underline{\hspace{1cm}}$$

$$\delta_{Ad} = |(hw - \pi d^2/4) - \underline{\hspace{2cm}} \underline{\hspace{1cm}}| = \underline{\hspace{2cm}} \underline{\hspace{1cm}}$$

$$\delta_A = (\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2)^{1/2} = \underline{\hspace{2cm}} \underline{\hspace{1cm}}$$

You should quote your value for A in the form  $A \pm \delta_A$  (units):  $\underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$   
( $\delta_A$  is normally given with one or at most two significant figures while the most significant figure in the value of A should be determined from  $\delta_A$ , as in  $A = 3.65 \pm 0.03 \text{ cm}^2$ . See Appendix V, Section D.)

## DERIVATIVE METHOD

(Optional for P115 students)

$$\delta_{Ah} = \left| \frac{\partial A}{\partial h} \right| \delta_h = \left| \frac{\partial}{\partial h} \left( \frac{\pi d^2}{4} - hw \right) \right| \delta_h = w \delta_h = \underline{\hspace{2cm}} \underline{\hspace{1cm}} \text{ (units)}$$

$$\delta_{Aw} = \left| \frac{\partial A}{\partial w} \right| \delta_w = \left| \frac{\partial}{\partial w} \left( \frac{\pi d^2}{4} - hw \right) \right| \delta_w = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\delta_{Ad} = \left| \frac{\partial A}{\partial d} \right| \delta_d = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\delta_A = \sqrt{\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2} = \underline{\hspace{2cm}} \underline{\hspace{1cm}}$$

$$A = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$$

You should find that the computational and derivative methods give similar results.

**GRADE:**                       
(out of 10 points)

**GRADED BY**                       
(TA's initials)