

Inclined Plane

Department of Physics, Case Western Reserve University
Cleveland, OH 44016-7079

Abstract:

I have used a system of a cart on an inclined plane connected to a counterweight by a string over a pulley to test Newton’s Second Law of Motion. Newton’s Second Law predicts an acceleration of the system of $a_{pred} = \text{_____} \pm \text{_____}$. After releasing the system from rest, I measured the acceleration of the system as $a_{meas} = \text{_____} \pm \text{_____}$. This measured acceleration is not consistent with the predicted acceleration. _____

(If your measured accelerations are consistent with the predicted acceleration, cross out the “not” preceding “consistent.” Give a one-sentence conclusion about the lab.)

Theory and Background:

One can determine the acceleration of the system depicted in Figure 1 by using Newton’s Second Law to analyze the motion. Assuming that the frictional force \vec{f} is negligible and that the pulley is massless and frictionless, the acceleration a of the system is¹

_____ . (1)

where _____

(Write down the appropriate equation; define all variables that haven’t been defined yet.)

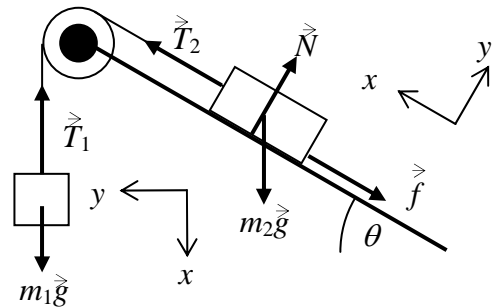


Figure 1: Schematic of Forces in Experiment. Courtesy Schultz, 2004.

One can apply Eq. 1 to find the sine of the angle of the incline. If we adjust m_1 and m_2 so that the acceleration is zero, then

_____ (2)

\Rightarrow _____ (3)

\Rightarrow _____ (4)

(Eq. 2 should be Eq. 1 with $a = 0$; Eq. 3 should be an intermediate algebra step; Eq. 4 should be $\sin\theta$ in terms of m_1 and m_2 .)

Equation 1 also implies that velocity as a function of time will be

_____ (5)

where _____.

From Eq. 5, we can see that if we fit a straight line to a plot of v vs. t , the slope of the line will be the acceleration and the intercept will be the velocity at time zero.

Procedure:

I measured m_2 with an electronic balance and determined that $\text{___} = \text{___} \pm \text{___}$.

I estimated the uncertainty δ_{m_2} as ___ because _____.

To get an estimate of the angle of the incline I estimated the length L and height H of the incline as in Figure 2. I used _____ to measure $H = \text{___}$ and $L = \text{___}$. Getting accurate and precise measurements of H and L were difficult because _____.

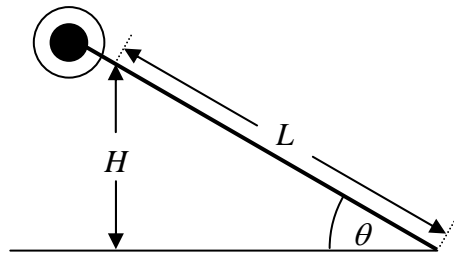


Figure 2: Estimating the angle. Courtesy Schultz, 2004.

I compensated for these difficulties by _____
 _____.

Because of these issues, I estimate that my uncertainty in H is $\delta_H = \text{_____}$ and my uncertainty in L is $\delta_L = \text{_____}$.

My first estimate of $\sin\theta$ is then

$$\sin\theta = \text{_____} \tag{6}$$

$$\Rightarrow \sin\theta = \text{_____} = \text{_____}$$

(Put the appropriate variables in the first line, put your actual measurements and final value for $\sin\theta$ in the second line.)

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta_{\sin\theta,H}^2 + \delta_{\sin\theta,L}^2} \tag{7}$$

where $\delta_{\sin\theta,H}$ is the uncertainty in $\sin\theta$ due to δ_H and $\delta_{\sin\theta,L}$ is the uncertainty in $\sin\theta$ due to δ_L .

Using the “computational method” to determine $\delta_{\sin\theta,H}$ and $\delta_{\sin\theta,L}$, I obtain

$$\delta_{\sin\theta,H} = \text{_____} = \text{_____} \tag{8}$$

$$\Rightarrow \delta_{\sin\theta,H} = \text{_____} = \text{_____}$$

and

$$\delta_{\sin\theta,L} = \text{_____} \tag{9}$$

$$\delta_{\sin\theta,L} = \text{_____} = \text{_____},$$

yielding $\delta_{\sin\theta} = \sqrt{(\text{_____})^2 + (\text{_____})^2} = \text{_____}$, or $\sin\theta = \text{_____} \pm \text{_____}$. Since $\theta =$

$\sin^{-1}(\sin\theta)$, the uncertainty in θ is

$$\delta\theta = \text{_____} - \text{_____} \quad (10)$$

$$\Rightarrow \delta\theta = \text{_____} - \text{_____} = \text{_____},$$

or $\theta = \text{_____} \pm \text{_____}$. I now used this estimate of θ to determine an estimate of the mass m_1 required to balance the system by taking Eq. 4 and solving for m_1 :

$$m_1 = \text{_____} \quad (11)$$

$$\Rightarrow m_1 = \text{_____} = \text{_____}.$$

(Solve Eq. 4 for m_1 , substitute in the appropriate numbers, and solve.)

I then set the mass of m_1 to _____ by adding masses to the hanger. After releasing the cart, the system was not in balance; the system _____.

(If the system was balanced, cross out “not;” describe the system’s motion in the blank.)

In order to obtain a better estimate of the angle of the incline and to account for the small amount of friction in the system, I then found the minimum and maximum masses that lead to zero acceleration (m_{\min} and m_{\max}) by adding and removing mass from m_1 . _____

(State if you tested zero acceleration by a stationary cart or cart moving with constant velocity. If you used the constant velocity test, also state how you determined the cart was moving at constant speed. Write a sentence about your reasons for choosing your methods, i.e., the advantages and disadvantages of your methods over other choices.)

Using the procedure above, I determined that $m_{\min} = \text{_____}$ and $m_{\max} = \text{_____}$, each with negligible uncertainty. I then set the average of m_{\min} and m_{\max} to be the “balancing mass” m_0 and half the difference between m_{\min} and m_{\max} as the uncertainty δ_{m_0} , so $m_0 = \text{_____} \pm \text{_____}$.

Substituting m_0 in for m_1 in Eq. 4, we see that sine of the angle of the incline is

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{height}}{\text{length}} .$$

The uncertainty in $\sin \theta$ is

$$\delta_{\sin \theta} = \sqrt{\delta_{\sin \theta, m_0}^2 + \delta_{\sin \theta, m_2}^2} \tag{12}$$

where $\delta_{\sin \theta, m_0}$ is the uncertainty in $\sin \theta$ due to δ_{m_0} and $\delta_{\sin \theta, m_2}$ is the uncertainty in $\sin \theta$ due to

δ_{m_2} . Using the “computational method” to determine $\delta_{\sin \theta, m_0}$ and $\delta_{\sin \theta, m_2}$, I obtain

$$\delta_{\sin \theta, m_0} = \frac{\delta_{m_0}}{m_0} \sin \theta = \frac{\delta_{m_0}}{m_0} \frac{h}{L} \tag{13}$$

$$\Rightarrow \delta_{\sin \theta, m_0} = \frac{\delta_{m_0}}{m_0} \frac{h}{L}$$

and

$$\delta_{\sin \theta, m_2} = \frac{\delta_{m_2}}{m_2} \sin \theta = \frac{\delta_{m_2}}{m_2} \frac{h}{L} \tag{14}$$

$$\delta_{\sin \theta, m_2} = \frac{\delta_{m_2}}{m_2} \frac{h}{L} ,$$

yielding $\delta_{\sin \theta} = \sqrt{\left(\frac{\delta_{m_0}}{m_0} \frac{h}{L}\right)^2 + \left(\frac{\delta_{m_2}}{m_2} \frac{h}{L}\right)^2} = \frac{h}{L} \sqrt{\left(\frac{\delta_{m_0}}{m_0}\right)^2 + \left(\frac{\delta_{m_2}}{m_2}\right)^2}$, or $\sin \theta = \frac{h}{L} \pm \delta_{\sin \theta}$.

(Compare this value of $\sin \theta$ with the value from direct measurement of L and H .)

I will adopt this value for $\sin \theta$.

I then set the counterweight m_1 to a value of _____ g to allow the cart to accelerate up the plane. I used an encoded pulley and *Logger Pro* software to record the motion of the cart.²

Results:

The average acceleration recorded by *Logger Pro* was $a_{\text{meas1}} = \text{_____} \pm \text{_____}$. We also measured the acceleration by applying Eq. 5 to a plot of the system’s velocity versus time.

See Figure 3. (*Attach a copy of your v vs. t graph labeled “Figure 3” to the end of the report.*)

The slope of the graph is $a_{\text{meas}2} = \text{_____} \pm \text{_____}$. These two values _____
(*agree/do not agree to within their uncertainties/are exactly the same.*) $a_{\text{meas}2}$ should be more
accurate because _____
and I will adopt it as the measured value, a_{meas} .

One can determine a theoretical value for the acceleration of the system, a_{pred} , by
substituting in known values into Eq. 1. Since I did not measure θ directly, I will substitute Eq. 4
(with m_1 as m_0 , the “balancing mass”) into Eq. 1 to obtain

$$a_{\text{pred}} = \text{_____} \tag{15}$$

$$\Rightarrow a_{\text{pred}} = \text{_____} = \text{_____}.$$

Error Analysis:

To find the uncertainty in a_{pred} , $\delta_{a_{\text{pred}}}$, I must find the contribution to $\delta_{a_{\text{pred}}}$ for each of the
quantities in Eq. 15 and add them in quadrature. The uncertainties in _____ are
negligible compared to the other quantities because _____
_____.

*(Identify any quantities that you will treat as having negligible uncertainty and justify your
treatment. Then show your work in estimating the uncertainty in a_{pred} .)*

(State whether or not your values agree within their uncertainties. If they do not agree, suggest sources of systematic error that were not adequately accounted for. Make a quantitative statement about the effect friction should have had on this experiment. Make a conclusion about Newton's Second Law.)

Acknowledgements:

I would like to thank _____, CWRU Department of Physics, for _____ help in obtaining the experimental data and preparing the figures. _____

(Thank your lab partner(s). If they or anyone else gave you additional assistance, say who they were and specifically what their assistance was.)

References:

(If you have any additional references, list them below. Make sure to indicate with an endnote where in the report you referred to the reference.)

1. Schultz, D., *General Physics I: Mechanics Lab Manual*, "Inclined Plane," CWRU Bookstore, 2004.
- 2.

End Notes:

¹ Schultz, D., p. 2.
² Schultz, D., p. 3, describes the encoded pulley.